Cluster Algebras and Statistical Physics
August 15-19, 2011
ABSTRACTS

About lattice integrable systems and cluster algebras
Rinat Kedem, UIUC

A brief, selective history of the origin of several functional equations arising in lattice integrable systems, in particular, T, Q and Y systems. I'll describe some approaches to their solutions, and the cluster algebras associated with them. I will point out several senses in which integrability plays a crucial role in these equations.

Non-Commutative Integrability, Paths and Quasi-determinants: Towards Non-Commutative Cluster Algebras
P. Di Francesco (Institut de Physique Theorique, CEA Saclay, France)

Classical Q-systems arising from the representation theory of quantum spin chains are part of some natural cluster algebras. Here we describe a non-commutative version thereof, based on the Q-system path model solutions. The main ingredient is discrete integrability, namely the existence of conserved quantities modulo the Q-system. In the case of rank 2, this leads to a natural general definition of non-commutative cluster algebra. For higher rank, we find that the Q-system must be replaced by a discrete non-commutative Hirota equation involving Gelfand-Retakh quasi-determinants of discrete Wronskian matrices.
(Joint work with Rinat Kedem, Advances in Mathematics 228 (2011) 97152).

On tropical dualities in cluster algebras
Andrei Zelevinsky, Northeastern University

In a joint work with T. Nakanishi, we study two families of integer vectors playing a crucial part in the structural theory of cluster algebras: the g-vectors parameterizing cluster variables, and the c-vectors parameterizing the coefficients. We prove two identities relating these vectors to each other. The proofs depend on the sign-coherence assumption for c-vectors that still remains unproved in full generality. If time allows, we discuss recent results due to D.Speyer and H.Thomas (work in progress), sharpening the sign-coherence property.

T-systems, Y-systems, and cluster algebras
Tomoki Nakanishi, Nagoya University

In 90's the systems of discrete functional equations called T-systems and Y-sytems were introduced and studied in the Bethe ansatz method for integrable models. After the introduction of cluster algebras by Fomin and Zelevinsky around 2000 it has been gradually recognized that T-systems and Y-sytems are a part of cluster algebra structure. In particular the long standing conjecture of periodicities of Y-systems by Zamolodchikov et al. is proved by the tropicalization method in cluster algebras. One can associate classical and quantum dilogarithm identities with any period of a cluster algebra. As a further consequence, the long standing conjecture of the central charge identities in conformal field theory by Kirillov et al. is proved.

Singularities of the pentagram map
Max Glick, University of Michigan

The pentagram map, introduced by R. Schwartz, is defined by the following construction: given a polygon as input, draw all of its "shortest" diagonals, and output the smaller polygon which they cut out. There exists a set of coordinates on the space of polygons, given by cross-ratios, which transform under the pentagram map according to the Y-dynamics of a cluster algebra. We study the singularities of the pentagram map. In particular, we show that a typical singularity disappears after a finite number of iterations. Additionally, we provide a method of moving past such a singularity by constructing the first subsequent iterate that is defined.

Cluster algebras and discrete integrable systems
Michael Shapiro, Michigan State

In this talk we discuss planar network construction that allows us to reproduce some results for pentagram map. Introduced by R. Schwartz about 20 years ago, the pentagram map acts on plane n-gons by drawing the diagonals that connect second-nearest vertices and taking the new n-gon formed by their intersections. It was shown recently by R. Schwartz, V. Ovsienko, S. Tabachnikov that the pentagram map, in particular, is completely integrable. M. Glick showed relation between pentagram map and the theory of cluster algebras. We will discuss these results from the point of view of general theory of networks. We also will discuss generalizations of pentagram map. This is a joint project with M. Gekhtman, S. Tabachnikov, and A. Vainshtein.

Integrability and entropy in cluster maps
Andrew N. W. Hone, University of Kent

We explain how symplectic maps natural arise from cluster algebras defined by quivers with a certain mutation periodicity property which was introduced recently by Fordy and Marsh. Many famous recurrences arise in this way, in particular the Somos-4 recurrence which also appeared in the hard hexagon model of statistical mechanics. We describe what it means for these maps to be integrable, and present a classification of them based on the notion of algebraic entropy.

Dimers and clusters
Rick Kenyon, Brown University

This is joint work with A. Goncharov. To any convex integer polygon we associate a cluster variety, which is essentially the moduli space of connections on line bundles on bipartite graphs on a torus. There is an underlying integrable Hamiltonian system whose Hamiltonians are sums of dimer covers.

A combinatorial description of the totally nonnegative Grassmannian
Kelli Talaska, UC Berkeley

The totally nonnegative Grassmannian is the subset of the Grassmannian in which all Pl¸cker coordinates have the same sign. As with totally positive matrices, it is possible to provide a parametrization using certain weighted planar graphs, and the weights can be written as Laurent monomials in the variables of a particularly nice cluster. We will give combinatorial formulas (in both directions) for a bijection between points in the TNN Grassmannian and these special graphs.

KP solitons and cluster algebras
Yuji Kodama, Ohio State

We start with a realization of the totally non-negative (tnn) part of the Grassmannian Gr(N,M) in terms of the soliton solutions of the KP equation. We then construct a decomposition of the tnn part of the Grassmannian according to the "asymptotic" spatial pattern of the soliton solutions. This leads to a classification theorem of all solitons solutions of the KP equation, showing that each soliton solution is uniquely parametrized by a derrangement of the symmetric group S_M. Each derangement defines the so-called Le-diagram of Postnikov. Then we show that the Le-diagram provides a complete classification of the "entire" spatial patterns of the soliton solutions coming from the tnn part of Gr(N,M) for asymptotic values f the time. We also classify the spatial patterns of the soliton solutions from the totally positive part of Gr(2,M), and give a soliton interpretation of the cluster algebra structure of A-type. In particular, we construct the associahedron in the space of multi-times of the KP hierarchy where each time represents the flow parameter of symmetry of the KP equation. This talk is based on a joint work with Lauren Williams.

Quiver mutation and quantum dilogarithm identities
Bernhard Keller, Jussieu

Quiver mutation is an elementary operation on quivers which appeared in physics in Seiberg duality in the 1990s and in mathematics in Fomin-Zelevinsky's definition of cluster algebras in 2002. In this talk, I will show how, by comparing sequences of quiver mutations, one can construct identities between products of quantum dilogarithm series. These identities generalize Faddeev-Kashaev-Volkov's classical identity and the identities obtained recently by Reineke. Morally, the new identities follow from Kontsevich-Soibelman's theory of refined Donaldson-Thomas invariants. They can be proved rigorously using the theory linking cluster algebras to quiver representations.

Belavin-Drinfeld classification and cluster structures on simple Lie groups
Michael Gekhtman, Notre Dame

We study natural cluster structures in the rings of regular functions on simple complex Lie groups, and Poisson-Lie structures compatible with these cluster structures. According to our main conjecture, each class in the Belavin-Drinfeld classification of Poisson-Lie structures on G corresponds to a cluster structure in O(G). I will explain how different parts of the conjecture are related to each other and present a supporting evidence that includes SL(n), n<5, Cremmer-Gervais Poisson bracket on SL(n), as well as the standard Poisson-Lie structure on any simple G. This is a joint project with M. Shapiro and A. Vainshtein.

Positivity for cluster algebras from surfaces
Ralf Schiffler, University of Connecticut

We present a combinatorial expansion formula for the cluster variables of cluster algebras that are associated to surfaces. The formula is given in terms of perfect matchings of a graph associated to the cluster variable, and it shows the positivity of the coefficients in the expansion. This is a joint work with G. Musiker and L. Williams.

Cluster Algebras of Surfaces II: Towards Bases
Gregg Musiker, University of Minnesota

In previous work with Ralf Schiffler and Lauren Williams, we have provided combinatorial formulas, involving snake graphs, for Laurent expansions for cluster variables coming from a bordered surface. In this talk, I will discuss extensions of this work towards constructing vector space bases for such algebras. This is work in progress with Schiffler and Williams.

n-to-1 Graphs, Discrete Unsolvability of the Inverse Problem
James Morrow, University of Washington

This talk will describe graphs for which the inverse problem for electrical networks is discretely unsolvable. The graphs have a finite set of conductivities that give the same Dirichlet-to-Neumann map. This is work of Chad Klumb and Courtney Kempton.

Title TBA
Dylan Thurston, Columbia

Flow polytopes and the Kostant partition function
Karola Meszaros, MIT/University of Michigan

This talk will examine several aspects of the connection between the Kostant partition function and the combinatorics of flows. First I will show how to use flows to establish Kostant partition function identities. Then I will describe joint work with Alejandro Morales generalizing an intriguing theorem of Stanley and Postnikov, which expresses the volume of a flow polytope as a Kostant partition function. Finally, I will present a method for constructing flow polytopes with combinatorial volumes, such as the number of r-ary trees on n internal vertices. Catalan numbers will also make an appearance.

Explicit expressions for cluster variables
Kyungyong Lee, University of Connecticut

We report an ongoing joint project with Ralf Schiffler. Let A be any cluster algebra with skew-symmetric exchange matrices, and let x be any choice of initial cluster. We show that the Laurent expansions of a large class of cluster variables with respect to the cluster x have nonnegative coefficients. (joint work with Ralf Schiffler)